Worksheet

12/07/2020

Free on dw-math.com

Problem quickname: 3339

<u>1)</u>

Solve the linear equation system. Use the Gaussian Elimination algorithm.

a)
$$
\begin{array}{rcrcrcrcr}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 \\
4x_1 &+& -14x_2 &+& 19x_3 &+& 4x_4 &=& 66 \\
24x_1 &+& 6x_2 &+& -31x_3 &+& 50x_4 &=& -164 \\
8x_1 &+& 26x_2 &+& -32x_3 &+& 20x_4 &=& -220
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
4x_1 &+& -14x_2 &+& 19x_3 &+& 4x_4 &=& 66 & (2) & |+(1) \\
24x_1 &+& 6x_2 &+& -31x_3 &+& 50x_4 &=& -164 & (3) \\
8x_1 &+& 26x_2 &+& -32x_3 &+& 20x_4 &=& -220 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
 && -6x_2 &+& 10x_3 &+& -2x_4 &=& 38 & (2) \\
24x_1 &+& 6x_2 &+& -31x_3 &+& 50x_4 &=& -164 & (3) & |+6 \text{ x }(1) \\
8x_1 &+& 26x_2 &+& -32x_3 &+& 20x_4 &=& -220 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
 && -6x_2 &+& 10x_3 &+& -2x_4 &=& 38 & (2) \\
 && 54x_2 &+& -85x_3 &+& 14x_4 &=& -332 & (3) \\
8x_1 &+& 26x_2 &+& -32x_3 &+& 20x_4 &=& -220 & (4) & |+2 \text{ x }(1)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
 && -6x_2 &+& 10x_3 &+& -2x_4 &=& 38 & (2) \\
 && 54x_2 &+& -85x_3 &+& 14x_4 &=& -332 & (3) & |+9 \text{ x }(2) \\
 && 42x_2 &+& -50x_3 &+& 8x_4 &=& -276 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
 && -6x_2 &+& 10x_3 &+& -2x_4 &=& 38 & (2) \\
 && && 5x_3 &+& -4x_4 &=& 10 & (3) \\
 && 42x_2 &+& -50x_3 &+& 8x_4 &=& -276 & (4) & |+7 \text{ x }(2)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
 && -6x_2 &+& 10x_3 &+& -2x_4 &=& 38 & (2) \\
 && && 5x_3 &+& -4x_4 &=& 10 & (3) \\
 && && 20x_3 &+& -6x_4 &=& -10 & (4) & |+(-4) \text{ x }(3)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrll}
-4x_1 &+& 8x_2 &+& -9x_3 &+& -6x_4 &=& -28 & (1) \\
 && -6x_2 &+& 10x_3 &+& -2x_4 &=& 38 & (2) \\
 && && 5x_3 &+& -4x_4 &=& 10 & (3) \\
 && && && 10x_4 &=& -50 & (4)
\end{array}
$$

Determine variable values:

$(4) \Rightarrow x_4 = -5$

$(3) \Rightarrow 5x_3 + 20 = 10 \Rightarrow 5x_3 = -10 \Rightarrow x_3 = -2$

$(2) \Rightarrow (-6)x_2 + (-20) + 10 = 38 \Rightarrow (-6)x_2 = 48 \Rightarrow x_2 = -8$

$(1) \Rightarrow (-4)x_1 + (-64) + 18 + 30 = (-28) \Rightarrow (-4)x_1 = -12 \Rightarrow x_1 = 3$

Solution: $x_1 = 3, x_2 = -8, x_3 = -2, x_4 = -5$

b)

$$
\begin{array}{rcrcrcrcr}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 \\
-63x_1 & + & -64x_2 & + & -78x_3 & + & -15x_4 & = & 452 \\
-35x_1 & + & -96x_2 & & & + & 34x_4 & = & -51 \\
-7x_1 & + & -64x_2 & + & 44x_3 & + & 58x_4 & = & -127
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
-63x_1 & + & -64x_2 & + & -78x_3 & + & -15x_4 & = & 452 & (2) \quad | + (-9) \times (1) \\
-35x_1 & + & -96x_2 & & & + & 34x_4 & = & -51 & (3) \\
-7x_1 & + & -64x_2 & + & 44x_3 & + & 58x_4 & = & -127 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
& & 8x_2 & + & -6x_3 & + & -6x_4 & = & 38 & (2) \\
-35x_1 & + & -96x_2 & & & + & 34x_4 & = & -51 & (3) \quad | + (-5) \times (1) \\
-7x_1 & + & -64x_2 & + & 44x_3 & + & 58x_4 & = & -127 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
& & 8x_2 & + & -6x_3 & + & -6x_4 & = & 38 & (2) \\
& & -56x_2 & + & 40x_3 & + & 39x_4 & = & -281 & (3) \\
-7x_1 & + & -64x_2 & + & 44x_3 & + & 58x_4 & = & -127 & (4) \quad | + (-1) \times (1)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
& & 8x_2 & + & -6x_3 & + & -6x_4 & = & 38 & (2) \\
& & -56x_2 & + & 40x_3 & + & 39x_4 & = & -281 & (3) \quad | + 7 \times (2) \\
& & -56x_2 & + & 52x_3 & + & 59x_4 & = & -173 & (4)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
& & 8x_2 & + & -6x_3 & + & -6x_4 & = & 38 & (2) \\
& & & & -2x_3 & + & -3x_4 & = & -15 & (3) \\
& & -56x_2 & + & 52x_3 & + & 59x_4 & = & -173 & (4) \quad | + 7 \times (2)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
& & 8x_2 & + & -6x_3 & + & -6x_4 & = & 38 & (2) \\
& & & & -2x_3 & + & -3x_4 & = & -15 & (3) \\
& & & & 10x_3 & + & 17x_4 & = & 93 & (4) \quad | + 5 \times (3)
\end{array}
$$

$$
\begin{array}{rcrcrcrcrl}
-7x_1 & + & -8x_2 & + & -8x_3 & + & -x_4 & = & 46 & (1) \\
& & 8x_2 & + & -6x_3 & + & -6x_4 & = & 38 & (2) \\
& & & & -2x_3 & + & -3x_4 & = & -15 & (3) \\
& & & & & & 2x_4 & = & 18 & (4)
\end{array}
$$

Determine variable values:

$(4) \Rightarrow x_4 = 9$
$(3) \Rightarrow (-2)x_3 + (-27) = (-15) \Rightarrow (-2)x_3 = 12 \Rightarrow x_3 = -6$
$(2) \Rightarrow 8x_2 + 36 + (-54) = 38 \Rightarrow 8x_2 = 56 \Rightarrow x_2 = 7$
$(1) \Rightarrow (-7)x_1 + (-56) + 48 + (-9) = 46 \Rightarrow (-7)x_1 = 63 \Rightarrow x_1 = -9$

Solution: $x_1 = -9, x_2 = 7, x_3 = -6, x_4 = 9$

<u>2)</u>

Solve the linear equation system. Use the Gaussian Elimination algorithm.

a)
$$\begin{array}{rcrcrcr} -8a & + & 2b & + & 10c & = & 60 \\ 56a & + & -17b & + & -72c & = & -410 \\ & & & & 10c & = & 40 \end{array}$$

$$\begin{array}{rcrcrcll} -8a & + & 2b & + & 10c & = & 60 & (1) \\ 56a & + & -17b & + & -72c & = & -410 & (2) \quad | + 7 \text{ x } (1) \\ & & & & 10c & = & 40 & (3) \end{array}$$

$$\begin{array}{rcrcrcll} -8a & + & 2b & + & 10c & = & 60 & (1) \\ & & -3b & + & -2c & = & 10 & (2) \\ & & & & 10c & = & 40 & (3) \end{array}$$

Determine variable values:

$(3) \Rightarrow x_3 = 4$
$(2) \Rightarrow (-3)x_2 + (-8) = 10 \Rightarrow (-3)x_2 = 18 \Rightarrow x_2 = -6$
$(1) \Rightarrow (-8)x_1 + (-12) + 40 = 60 \Rightarrow (-8)x_1 = 32 \Rightarrow x_1 = -4$

Solution: $a = -4, b = -6, c = 4$

b)
$$\begin{array}{rcrcrcr} -3a & + & 3b & + & -6c & = & -51 \\ -24a & + & 31b & + & -51c & = & -462 \\ & & -63b & + & 19c & = & 454 \end{array}$$

$$\begin{array}{rcrcrcll} -3a & + & 3b & + & -6c & = & -51 & (1) \\ -24a & + & 31b & + & -51c & = & -462 & (2) \quad | + (-8) \text{ x } (1) \\ & & -63b & + & 19c & = & 454 & (3) \end{array}$$

$$\begin{array}{rcrcrcll} -3a & + & 3b & + & -6c & = & -51 & (1) \\ & & 7b & + & -3c & = & -54 & (2) \\ & & -63b & + & 19c & = & 454 & (3) \quad | + 9 \text{ x } (2) \end{array}$$

$$\begin{array}{rcrcrcll} -3a & + & 3b & + & -6c & = & -51 & (1) \\ & & 7b & + & -3c & = & -54 & (2) \\ & & & & -8c & = & -32 & (3) \end{array}$$

Determine variable values:

$(3) \Rightarrow x_3 = 4$
$(2) \Rightarrow 7x_2 + (-12) = (-54) \Rightarrow 7x_2 = -42 \Rightarrow x_2 = -6$
$(1) \Rightarrow (-3)x_1 + (-18) + (-24) = (-51) \Rightarrow (-3)x_1 = -9 \Rightarrow x_1 = 3$

Solution: $a = 3, b = -6, c = 4$

<u>3)</u>

Solve the linear equation system. Use the Gaussian Elimination algorithm.

a)
$$\begin{array}{rcrcl} 4y & + & -7z & = & 8 \\ 24y & + & -35z & = & 76 \end{array}$$

$$\begin{array}{rcrcll} 4y & + & -7z & = & 8 & (1) \\ 24y & + & -35z & = & 76 & (2) \quad | + (-6) \text{ x } (1) \end{array}$$

$$\begin{array}{rcrcll} 4y & + & -7z & = & 8 & (1) \\ & & 7z & = & 28 & (2) \end{array}$$

Determine variable values:

$(2) \Rightarrow x_2 = 4$
$(1) \Rightarrow 4x_1 + (-28) = 8 \Rightarrow 4x_1 = 36 \Rightarrow x_1 = 9$

Solution: $y = 9, z = 4$

b)
$$\begin{array}{rcrcl} 4y & + & 4z & = & -16 \\ -8y & + & -12z & = & 52 \end{array}$$

$$\begin{array}{rcrcll} 4y & + & 4z & = & -16 & (1) \\ -8y & + & -12z & = & 52 & (2) \quad | + 2 \text{ x } (1) \end{array}$$

$$\begin{array}{rcrcll} 4y & + & 4z & = & -16 & (1) \\ & & -4z & = & 20 & (2) \end{array}$$

Determine variable values:

$(2) \Rightarrow x_2 = -5$
$(1) \Rightarrow 4x_1 + (-20) = (-16) \Rightarrow 4x_1 = 4 \Rightarrow x_1 = 1$

Solution: $y = 1, z = -5$

<u>4)</u>

Solve the linear equation system.

a)
$$\begin{array}{rcrcl} -7y & + & -6z & = & 58 \\ -7y & + & -12z & = & 88 \end{array}$$

Triangular form:

$$\begin{array}{rcrcll} -7y & + & -6z & = & 58 & (1) \\ & & -6z & = & 30 & (2) \end{array}$$

Determine variable values:

$(2) \Rightarrow x_2 = -5$
$(1) \Rightarrow (-7)x_1 + 30 = 58 \Rightarrow (-7)x_1 = 28 \Rightarrow x_1 = -4$

Solution: $y = -4, z = -5$

b)
$$\begin{array}{rcrcr} -5y & + & 8z & = & 61 \\ 15y & + & -14z & = & -163 \end{array}$$

Triangular form:

$$\begin{array}{rcrcrl} -5y & + & 8z & = & 61 & (1) \\ & & 10z & = & 20 & (2) \end{array}$$

Determine variable values:

$(2) \Rightarrow x_2 = 2$
$(1) \Rightarrow (-5)x_1 + 16 = 61 \Rightarrow (-5)x_1 = 45 \Rightarrow x_1 = -9$

Solution: $y = -9, z = 2$

Good Luck!